

MATH WORKBOOK

Answers and Explanations

1. **E** Absolute value is a positive number: $2(7) - 3(7) = 14 - 21 = -7$
2. **C** The 0 doesn't repeat so that answer is automatically eliminated. $322/4 = 80.5$. That means the number would be FULLY repeated 80 times, and then $\frac{1}{2}$ of **another time**. Multiply $\frac{1}{2}$ by how many numbers (4) there are to get that it's the second place or the number 3.
3. N/A
4. **A** Watch signs and remember a minus sign followed by a negative sign makes a positive. $(4+2-(-5))(2+-5) = 11(-3) = -33$
5. **B** If using a calculator, separate mixed numbers into whole numbers and fractions and subtract one at a time. $6-1-\frac{1}{2} - \frac{3}{4} = 3\frac{3}{4}$
6. **B** This is a ratio where one shape is compared to another. Cross multiply the two numbers known and divide by the one that is unknown:

$$\frac{\text{length } 48}{\text{width } 32} = \frac{\text{length } 8}{\text{width } ?} \quad 32 \cdot 8 = 48x \quad x = 5\frac{1}{3}$$

7. **A** Quickly read the top portion and notice one important item: Enrollment by May 15th receives a 20% discount. Mr. Ramirez has one child in grade 4 (400) and one in grade 6 (500). That is a total of \$900. He receives a 20% discount. If he doesn't pay 20%, he does pay 80% (all percentages are based on 100% in the beginning). Thus, $900 \times .80 = \$720$
8. **H** $563 \text{ minutes} / 60 \text{ minutes} = 9 \text{ hours and } 23 \text{ minutes}$. Add 9 hours and 23 minutes to 4:51 to get 2:14 am. Just adding the 3 minutes in the one's place to the 1 in the one's place in the original time reveals that the new time will end in 4, so F and G are easily eliminated.

A Calculate the amount of time used driving and in the store before checking out. 1 hour 15 minutes + 25 minutes = 1 hour 40 minutes. Subtract 1 hour 40 minutes from 2:30 to get 12:50 pm
9. **C** The temperature drops 2 degrees per hour. Since there are 10 hours between 1 in the afternoon and 11 at night, that means the temperature drops 20 degrees ($2 \times 10 = 20$). The higher temperature is in the afternoon, so 20 needs to be added to 42 to find that the temperature at 1 pm is 62° .

10. **H** 18 miles in 20 minutes can be set up as a ratio to 60 minutes (as there are 60 minutes in an hour).

$$\frac{18 \text{ miles}}{20 \text{ minutes}} = \frac{? \text{ miles}}{60 \text{ minutes}} \qquad 20x = 18 \cdot 60 \quad x = 54$$

Since he did NOT exceed the speed limit, anything below 54 is eliminated. The speed limit must be 55.

D Recall that when the graph is straight, no one is moving or traveling, so those spots do not count as travel time. There are 6 hours of travel time which means driving only took place for 6 hours. The total distance traveled was 450 miles over 6 hours. Divide 450 by 6 to get 75.

11. **J** Multiplying two even numbers to get a product yields an even answer. 80 is the only even number answer.

A Every number has at least 2 factors, and other factors also come in pairs. However, perfect squares have an ODD number of distinct factors because the square root is only counted once. Thus, 16 has these factors: 16, 1, 8, 2, and 4 (only counted once).

12. **D** This is a least common multiple problem. Find where these two numbers come together with a product. Use the LCM button on your calculator (separate numbers with commas and enclosed in parentheses) or break each number down into prime numbers, circle where each unique number occurs the most, and multiply the circles.

$$6 = 3 \times 2$$
$$10 = 5 \times 2$$

Circle the 3, 5, and one 2 (occurs only once so circle either one). Multiply to get 30.

13. **K** Greatest common factor is the highest number that each number can be divided by evenly. Since both terms have an x, each number can be divided by an x. Thus, F and G are eliminated. Start with the last number since it is the greatest. Both numbers can be divided by 36, so the answer is 36x.

14. **B** To find the original total, multiply 160 x 3 to see that Juan, Jim, and Malik have a total weight of 480 lbs. When Harry is added in, the total is 600 (150 x 4). Subtract 600-480 to discover that Harry weighs 120 lbs.

15. **J** Outliers are “outside” the original data set. Consider that the outlier could be 1 or even 1000 (below or above the data set). If a number is above or below the data set, it changes the range since range is the greatest number minus the least number. Thus, any answer with range is eliminated (H and K). In addition, the mean will change as it’s the average of the numbers (F and G). Thus, only J is left.

$$\mathbf{J} \quad \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1)^2} = \frac{40320}{576} = 70$$

16. **J** Sam worked 40 hours at \$12 an hour which is \$480 for each week. Since he was paid \$570, he earned \$90 in overtime pay (570-480). He gets paid \$18 an hour in overtime pay, so he worked 5 hours overtime (90/18 = 5). He worked 40 regular hours and 5 overtime hours, for a total of 45 hours.

17. **B** Distribute the 3 on the left side to eliminate the parentheses first. Thus, $3x - 6 = 6x - 3$. Move the variables to one side and the numbers to the other (to undo an operation, perform the opposite operation, and what is done to one side must be done to the other) to get $-3 = 3x$. Divide both sides by 3 to get $-1 = x$. Then, substitute -1 into the $9x-2$ to get $(9)(-1) - 2 = -11$

18. **B** Since $\frac{1}{3}$ of the pieces are brown, $\frac{2}{3}$ of the pieces are the other colors. The remaining five colors are distributed evenly, so $\frac{2}{3}$ of the pieces divided by 5 gives the amount of pieces per color.

$\frac{2}{3} \div 5$ (to divide, multiply by the reciprocal of the second number)

$$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$$

19. **D** The number of spots is limited to 3. How many different plants can be placed in the first spot? 6. Once that place has a plant, only 5 plant choices are left. Thus, five plants are available and able to be chosen. Finally, for the third spot, only four plants are available to be used. Multiply the number of choices: $6 \times 5 \times 4 = 120$

20. **B** Use the calculator to multiply $3 \times .00000072$ to get .00000216. Move the decimal point to the right six places to get 2.16×10^{-6} . Since the answer is less than 0, the scientific notation is negative.

21. **E** Multiply the exponents since the exponent is being raised to a power. Multiply 3×21 to find 63.

22. **A** Choose a number for m that falls between 0 and 1 (e.g. .5). Use the calculator to raise .5 to the -1st power. Always eliminate the negative exponent by flipping it to the opposite side of the fraction.

$$.5^{-1} = \frac{1}{.5} = 2$$

- G** Choose any number (e.g. 7). Raise it to a negative power (e.g. -2).

$$7^{-2} = \frac{1}{49} = \text{a positive number.}$$

23. **D** Anything done in groups of 2 (two points on a line, games, shaking hands, dancing, etc.) can be calculated using the following formula:

$$\frac{N(N-1)}{2} = \frac{19(18)}{2} = 171$$

24. N/A

25. **H** Since the new total is 8 more than the original, take the difference (8) and divide by the total number of items (4) to get that the score increases by 2. The original average was 92, and the new one is 94.

E The original average is 85 (add 85, 86, 87, and 82 to get a total of 340; divide by 4 to get 85). Sofia wants the new average to be 3 points higher or 88. Multiply the total by 5 (88 x 5) to get 440. Subtract 440-340 to find the test score of 100.

26. **G** Isolate the x to one side to get that $7y + 5 = 2x$. Divide both sides by 2 to find that G is the answer.

G Isolate the I. Use the rules of solving equations to undo operations.

$$R = \frac{P}{I^2} \Rightarrow \text{Multiply both sides by } I^2 \Rightarrow RI^2 = P$$

$$\Rightarrow \text{Divide both sides by } R \Rightarrow I^2 = \frac{P}{R}$$

$$\Rightarrow \text{Take the square root of both sides to find the answer } I = \sqrt{\frac{P}{R}}$$

27. **D** Use unit conversions to cancel out units. Just like fractions can be canceled or reduced on the same side of the equal sign, so can units.

$$3898 \text{ steps} \times \frac{2.25 \text{ feet}}{1 \text{ step}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = 1.66$$

28. **J** Arithmetic sequences involve addition or subtraction. How many numbers are between the 10th number and the 1st? 9. Since the difference is -2 for each number, there is a difference of 18 (9×2) between the 10th and 1st number in the sequence. Subtract 18 from 30 to find the answer.

$$30 - 2(9) = 12$$

29. **C** A geometric sequence involves multiplication. Each answer shows 27 as the first term. The parentheses are the rate at which the sequence is multiplied. Since the numbers become greater (27...64), the numbers must be multiplied by a number greater than 1. Thus, A and B are eliminated as $\frac{3}{4}$ is less than 1. All geometric sequences are raised to the power of $n-1$, so C is the only possible answer.

30. **J** This is a system of equations but possible answers are given. Using total number of tickets sold (200 on the chart), the answers are as follows:

F. 22 Value Tickets (which means 178 premium tickets)

G. 50 Value Tickets (150 premium)

H. 100 Value (100 premium)

J. 150 Value (50 premium)

K 178 Value (22 premium)

All these numbers add up to the 200 total tickets sold. Start in the middle and plug in ticket prices ($100 \times 6 = \$600$ for value tickets and $100 \times 10 = \$1000$ for premium tickets). Since that total is too much, more value tickets need to be sold and less premium tickets (So F, G, and H are out). Plus in prices to J ($150 \times 6 = 900$ and $50 \times 10 = 500$). Since $900 + 500 = \$1400$.

31. **K** Functions are a fancy way of performing substitution. Use $f(x)$ and substitute it into the x in $g(x)$.

$$\frac{1}{4x + 12}$$

Since undefined includes having 0 in the denominator, set the denominator equal to 0 and solve. $4x + 12 = 0$ which means $4x = -12$ and thus, $x = -3$. So x cannot be -3 as that would cause the denominator to be 0.

D If $ac = 1$, neither a nor c is 0. If the fraction is undefined, then $d = 0$ (since 0 in the denominator is undefined). If $abc=d$, and a/c are not 0, then b must be equal to 0.

32. **G** Find the total number first by multiplying the number of students times the rating ($5 \times 0 + 1 \times 10 + 2 \times 25 + 3 \times 40 + 10 \times 4 = 220$). Divide 220 by the total number of students (90) to get $2\frac{4}{9}$.

33. **C** Solve as an equation (moving the x to the left side since the answers are also written in that format)

$$3 - 2x > 7 - x \Rightarrow -x > 4$$

\Rightarrow divide each term including the sign by -1 to get $x < -4$

34. **B** Perimeter means measuring around the outside. Remove the sidewalk by subtracting 6 from each number (3 on each side). $40 - 6 = 34$ and $30 - 6 = 24$. Add the dimensions around the subject.

35. **C** Circumference is $2\pi r$ for circles. Thus, the first circle is 10π and the second circle is 12π . The difference is $12\pi - 10\pi$ which is 2π .

36. **G** Figure the area of the rectangle which is 18×8 or 144. The area of each missing square is x times x or x^2 . There are four missing small squares which is 4 times x^2 or $4x^2$. Subtract the squares from the total which is $144 - 4x^2$.

D The carpet squares are in inches, and the question asks for the number of carpet squares, so convert everything to inches first. Since the room is $8\frac{1}{3}$ feet by 10 feet, that's the same as 100 inches by 120 inches or 12000 inches². Each carpet tile is 20 inches by 20 inches which is 400 inches². Divide 12000 by 400 to get 30 carpet squares.

37. **D** Surface area is always the outside surface (both bases and the lateral surface area around the middle or how much wrapping paper would be used to cover the surface of the shape) of the shape. The formula for surface area for all right cylinders, prisms, and cubes is $2B_A + P_B H$ (B_A is the area of the base, P_B is the perimeter of the base, and H is the height of the shape).

The base of this shape is a rectangle that is 4×2 which is 8. The perimeter of the same base is $4 + 2 + 4 + 2$ which is 12. The height of the shape is 3 (the line that connects the base to the base). Input the numbers into the formula:

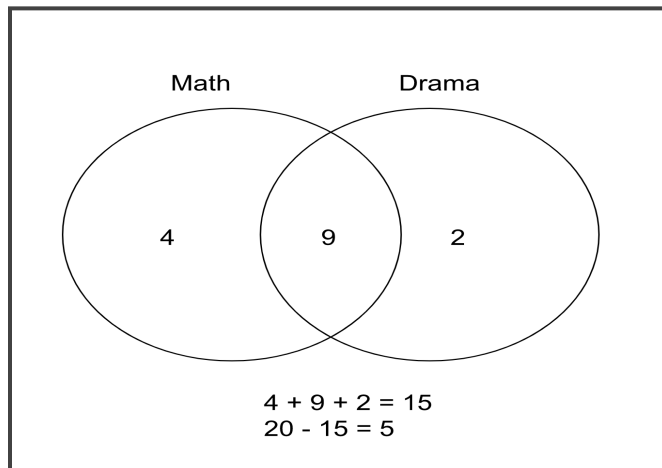
$2(8) + (12)(3) = 52$. Since the recycling is $.01/\text{lb}$ (not shown), the cost of the box is $52 \times .01$ or $.52$.

38. **F** The volume of a right rectangular prism is BH (area of the base \times the height). Since the volume is given, and the base does not change, only the height is missing.

$$175 \text{ ft}^3 = 25 \text{ (area of the base)} \times \text{Height}$$

Solve for the height by dividing both sides by 25 to determine that the height is 7.

39. **G** Draw a Venn diagram and write the shared number in the middle. Subtract the shared number from the total number in each group to determine exactly how many people are in math, both, and drama. Then, subtract the total in those groups from the total number of students.



40. **G** Amplitude is how high the cosine or sine graph extends. It is always the first number in the equation of the graph!
41. **H** Midpoint is the average of the x's and the average of the y's. In this question, the midpoint and one end are known. However, the other end is unknown. Substitute into the equation for midpoint.

$$\frac{1+x}{2} = 3 \text{ and } \frac{2+y}{2} = 5$$

Multiply both sides by the denominator to get $1+x = 6$ and $2+y = 10$
Solve to find that $x = 5$, and $y = 8$

42. **J** Use the distance formula to solve for distance.

$$\begin{aligned} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} &= \sqrt{\left(5\frac{1}{3} - 1\frac{5}{9}\right)^2 + (-3 - 3)^2} \\ &= 6\frac{8}{9} \end{aligned}$$

43. **H** Use the slope formula $(\frac{y_1 - y_2}{x_1 - x_2})$ to solve.

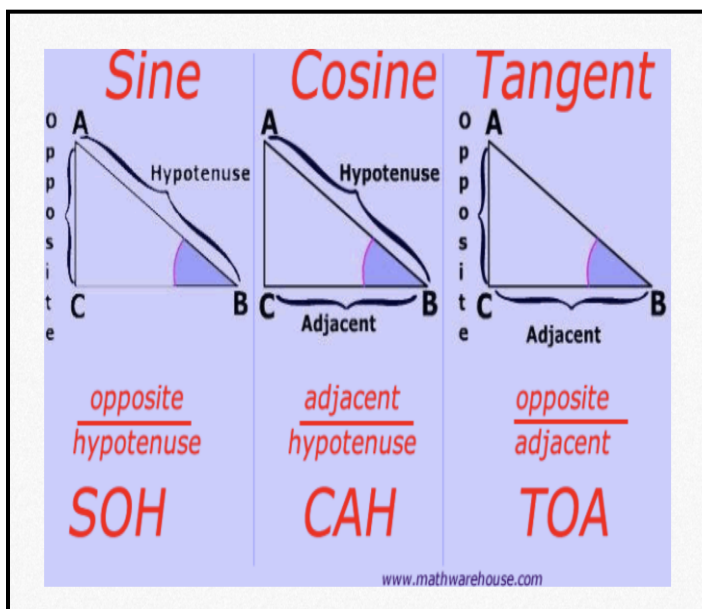
$$\frac{1-6}{-3-5} = \frac{-5}{-8} = \frac{5}{8}$$

44. **A** Since the slope is positive, the negative slopes of B, C, and E are eliminated. The y-intercept (where the graph crosses the y axis) is also positive, only A meets the criteria.

45. **G** Remember that reflect means to flip, translate means to slide, rotate means to turn, and dilate means to increase or decrease proportionally. When the F reflects across the X axis it flips to quadrant 3 (lower left). Then, it reflects again across the y axis to be in quadrant 4 (lower right) but in the direction of option K. Then, it rotates 90 degrees clockwise which places it back in quadrant 3 facing up which is option G.

46. **E** This is a 3-4-5 Pythagorean triple because AD (the hypotenuse) is 2500 (500 x 5), AB is 2000 (500 x 4), so BD must be 500 x 3 or 1500.

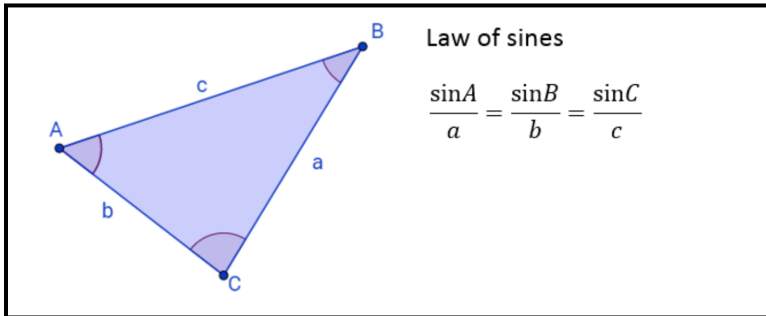
47. **G** Remember SOHCAHTOA! $\text{Sin} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{15}$



48. **B** The law of sines is a proportion. First figure the missing angle since the sum of all angles in a triangle equals 180. So $180 - 75 - 45 = 60^\circ$.

$$\frac{\sin 60^\circ}{240} = \frac{\sin 45^\circ}{\text{string}}$$

Use the calculator to cross multiply $(\sin 60)(\text{string}) = (\sin 45)(240) = 80\sqrt{6}$



K The law of cosines needs to be used here as there are not enough angles known to use the law of sines. Only the angle inside the triangle next to the 25° angle is known because it is $180-25$ or 155 . Thus, the law of cosines must be used, so answers F, G, and H are eliminated. Since only the 155° angle is known, and the angle is used in the cosine formula, the answer is K.

49. **K** Recall that vectors can be ANYWHERE on a graph. Only direction and length are vital. To calculate $u + v$, first determine the direction of each. U goes up 2 (+ 2) and to the right 1 (+1). V goes down 1 (-1) and to the right 1 (+ 1). Vectors can be placed ANYWHERE on the graph - just look for length and direction. Add the lengths: $2 + - 1 = 1$ (up 1). Add the directions: $1 + 1 = 2$ (to the right 1). The only vector that is going up 1 and to the right 2 is q.

50. **A** The amplitude is the first number in the equation for sine, so options D and E are eliminated. Look inside the parentheses to see that the period is 2. Since the normal period of a sine graph is 2π , the period only needs to be multiplied by π to achieve 2π .

K The amplitude of this graph (how high it goes on the y axis) is 3, so automatically F and G are eliminated. The period of the graph itself (how far between each crest is π (notice that the halfway point between the crests is $\frac{\pi}{2}$ so the whole period is π . Since the normal period of a sine graph is 2π , the period must be multiplied by 2 which is K.

51. **G** Drawing a line from point B down to the x axis yields a 45-45-90 triangle (based on the information given that angle BOC is 45° . That means that OB is the hypotenuse and that is given as 4 (the radii of the larger circle). Since a 45-45-90 triangle has a proportion of $x-x-x\sqrt{2}$, the hypotenuse is $x\sqrt{2}$.

$$4 = x\sqrt{2}. \text{ Solve by dividing each side by } \sqrt{2} \text{ to get } \frac{4}{\sqrt{2}}$$

B Since DEF is a right angle, this is a 30-60-90 triangle. That means that angle F is 60°. In the smaller EGF triangle, G is 90° and angle FEG is 30°. A 30-60-90 triangle yields an $x - x\sqrt{3} - 2x$ proportion. EF is across from the 90° angle, so 20 is $2x$ which makes GF (across from the 30° angle) x or 10. Thus, EG is $10\sqrt{3}$.

52. When given a coefficient outside the matrix, multiply the coefficient by each term in that matrix. Then add like terms (upper left added to upper left, etc.)

$$\begin{array}{r} 13 + 4(5) \\ 71 + 4(11) \end{array} \qquad \begin{array}{r} 4 + 4(3) \\ 3 + 4(22) \end{array} \qquad = \qquad \begin{array}{r} 33 \quad 16 \\ 115 \quad 91 \end{array}$$

K When adding or subtracting, pair matching locations (e.g. top left is 10 in the first matrix and top left in the second matrix is -9. $10 - (-9) = 19$)

Determinant: $AD - BC$

$$4(7) - 5(6) = -2$$

53. **J** Use the quadratic formula or solve by factoring to get $(x-9)(x+5)$. Since this is set equal to 0, x can be 9 or -5. Add those together to get 4.

G Factor the binomial to find $(x-9)(x+4)$.

Using a quadratic solver on the calculator also gives these answers. Remember the calculator will provide the answers/roots/solutions. The factors are the opposite signs of the answers.

54. **H** This equation means that x raised to some power (y) = 8. Since only 8^1 and $2^3 = 8$ and x must be less than 8, only 2^3 fits the requirements. Thus the exponent is 3.

A Assume the log base is 10 since it is not given. D is 100, so plug that into the formula. $100 = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$. First divide both sides by 10 to obtain

$10 = \log_{10}\left(\frac{I}{10^{-12}}\right)$. To get rid of the base, perform the opposite operation which is raising to a power of 10. Raise both sides to the power of 10 to get

$$10^{10} = \left(\frac{I}{10^{-12}}\right). \text{ Multiply both sides by } 10^{-12} \text{ to get } 10^{10} \times 10^{-12} = I \text{ or } I = 10^{-2}$$

55. **D** First, take -3 and substitute it into g (remember to work functions from the inside out). $(-3)^2 - 2 = 7$. Then, substitute 7 into $f(x)$ to get $5(7) + 1$ which is 36.

B Substitute $g(x)$ which is $x-3$ into $f(x)$ to get $(x-3)^2 + 1$. Multiply out $(x-3)(x-3) + 1$ to find that the expression equals $x^2 - 6x + 10$

56. **G** The equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$ where the center is h,k , and r is the radius. Since the radius is 5 and the radius needs to be squared in the equation of a circle, options F and J are out. If h, k is 4, -3 and those are substituted into the equation of the circle, they become $(x-4)^2 + (y-3)^2 = 5^2$ which is G.

57. **D** The equation of an ellipse is below with h,k as the center (like a circle), a is the minor axis (the line extending across the shortest part of the ellipse), and b is the major axis (the line extending across the longest part of the ellipse). Since a and b must be squared, options A, B, and C are eliminated. The center is at 2,-3 and can be found easily visually. Since the equation of an ellipse is similar on the top of the fraction, only D meets the requirements. In addition, the distance between the center and the side of the minor axis is 3 which means $a^2 = 9$, and the distance between the center and the side of the major axis is 5 which indicates that $b^2=25$.

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

58. **D** The foci (foe-sigh) of an ellipse includes 2 points equidistant from the center on the major axis. To find, subtract Major axis² - Minor axis² ($25-9$) to get the foci² (16). That makes the foci equal to 4. Find the center of the ellipse and count 4 away from the center in both directions on the major axis. Since the center of the ellipse is at (5,3) and the major axis is on the x axis (longer going left to right), count to the left 4 to get (1,3) and to the right 4 to get (9,3).

59. **G** With (-1,8) being the apex (and the center of the parabola), it is equidistant from (-3,0) and the point (a,0). Since the -1 is +2 away from -3, it is also 2 away from a. Add 2 to -1 to get 1.

60. **K** If x is less than -5, imagine it to be -6. When -6 is substituted under the radical sign, the number $(-\sqrt{-6 + 5})$ becomes $-\sqrt{-1}$. Since $i = \sqrt{-1}$, this is an

imaginary number (there is no such real number that is a negative under the radical sign).

E Only the complex conjugate of a number can eliminate imaginary numbers. Thus, $(3 + bi)(3 - bi)$ would yield $9 - bi^2$. Since $i^2 = -1$, the equation becomes $9 - (-1)b$ which eliminates the i in the equation.

61. **F** To move from degrees to radians, multiply by $\pi/180$.

$$30 \times \pi/180 = 30\pi/180 = \pi/6$$

D To convert from radians to degrees, multiply by $180/\pi$.

$$\frac{7\pi}{9} \times \frac{180}{\pi} = \frac{7(180)}{9} = 140^\circ$$

62. **G** To find the arc length:

- Find circumference - 24π
- Find the sector of the circle (how large is the portion of the circle). It is $45^\circ/360^\circ$ which is $\frac{1}{8}$ of the total circle.
- So $\frac{1}{8}$ of the circumference is 3π . Thus, this arc length is 9.42.
- For this question, the length of the zipper is the arc length (9.42) plus the radius (12) or 22.

63. **H** Since PS and PT are radii of the circle, they are congruent. Thus, angle S and angle T are both 30 degrees. Since angle T is on the circle, it is $\frac{1}{2}$ the measurement of the arc it creates (arc RS). Double the angle to find that arc RS is 60° .